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Introduction

The ability to perceive and process nonsymbolic numerosity plays a key role in the adaptation of the organism to the environment (Nieder, 2018). This ability emerged quite long ago during evolution, as numerosity sensitivity has been documented to some extent not only in humans and primates, but also in more evolutionarily ancient species such as amphibians or fish (Agrillo & Bisazza, 2018; Brannon, 2005). Researchers use a variety of terminology to describe the ability to perceive quantitative information: some prefer the term "Number Sense" (Agrillo & Bisazza, 2018; Burr & Ross, 2008; Dehaene, 2001), while others prefer "approximate number sense" (Halberda et al., 2008a), "intuitive number sense" (Feigenson et al., 2013), or "Nonsymbolic Number Sense" (Decarli et al., 2023). Despite some differences in terminology, most researchers agree that number sense involves the ability to compare sets of objects and select the set that contains a larger/fewer number (nonsymbolic comparison), to notice whether there have been changes in the number of objects (nonsymbolic detection of changes), and to establish quantitative similarities or differences by comparing two or more sets of objects (Berch, 2005; Gebuis & Van Der Smagt, 2011; Halberda et al., 2008a; Park et al., 2016; Sasanguie et al., 2014).

Research on the ability to process numerosity has developed into the 'number sense' theory, according to which the ability to process nonsymbolic quantitative information is provided by a separate nonsymbolic quantity representation system (Arrighi et al., 2014; Burr et al., 2018; Odic & Starr, 2018). This system is activated whenever a person is 'confronted' with quantitative information. In this case, 'numerosity' is described as a primary attribute of the perceived objects, along with other properties that can be processed in the early stages of perception: size, colour, shape (DeWind et al., 2019; Harvey & Dumoulin, 2017; Park et al., 2016).

Using various tasks (e.g. non-symbolic comparison tasks), researchers have found that one of the main features of the system of non-symbolic representation of quantity is its imprecision, its approximation (which is reflected in one of the widely used terms -

Approximate Number Sense). What is meant by this? First of all, no matter what tasks are used, accuracy never reaches 100%. For example, when comparing two sets of objects, a person will always make some mistakes, provided they compare them quickly and do not use counting.

It should be noted that there is also an accurate system for estimating nonsymbolic quantities, called subitising (Revkin et al., 2008). Subitising is the ability to quickly and accurately estimate the number of objects within 1-4 (1-3) (Anobile et al., 2019; Burr et al., 2010). More often, researchers consider subitising as a separate system (Feigenson et al., 2004; Revkin et al., 2008). In addition, they also distinguish a separate texture judgment system, the quantity representation system, when the number of objects is so large that the boundaries of individual objects are barely distinguishable (Anobile et al., 2014).

Returning to the system of non-symbolic representation of numerosity, it has been shown that representation errors obey certain regularities. In particular, the number of errors increases with response time as the size of the sets being compared gets closer, i.e. as the distance between them decreases and the numerosity ratio increases (Dehaene, 2003; Lyons et al., 2015). This means, for example, that a person is more likely to make an error when comparing 7 and 9 objects than when comparing 5 and 11 objects. This pattern has been called the 'numerical distance effect' or 'numerical proportion effect' (Dehaene, 2003; Dietrich et al., 2015; Lyons et al., 2015). Another pattern found when examining patterns of error making in nonsymbolic numerosity estimation or comparison is that the probability of error increases as numerosity increases (when numerical proportion remains intact). This pattern is called the 'size effect' (e.g. (Dehaene, 2001)).

The presence of these features is explained by the 'mental number line' model (Dehaene et al., 1993). This model assumes that perceived numerosities (numbers or sets of objects) are conventionally arranged along a line that runs from left to right (in cultures with corresponding writing patterns). Each perceived quantity corresponds to a specific population of neurons whose activation can be represented by a 'Gaussian' curve (Nuerk et al., 2011; Toomarian & Hubbard, 2018). When the numbers on this number line are far apart, the 'Gaussian' curves do not overlap, so each quantity corresponds to a separate representation. However, if the numbers are close together, the curves may overlap significantly, leading to an error in identifying the quantities and their relative locations on the mental number line.

In addition, the mental number line model may partly explain the relationship between the perception of quantity and the perception of space. For example, many studies have found the so-called SNARC (spatial-numerical association of response code) effect, one manifestation of which is that large numbers are associated with the right side of the visual field and small numbers with the left side (Chen & Verguts, 2010; Fischer et al., 2003; Nemeh et al., 2018).

The relationship between number and spatial perception is also supported by many neurophysiological studies. Number operations have been found to be associated with activation of posterior parietal areas that are also associated with spatial perception and attention (e.g. (Göbel et al., 2001; Göbel et al., 2006; Hubbard et al., 2005). Furthermore, it has been shown that there is a developmental change in the extent to which parietal areas are involved in quantitative information processing (Ansari et al., 2005). It has been shown that children show greater activation of the frontal zone, whereas adults show greater activation of the parietal zones.

Common neurophysiological correlates for the estimation of number and space gave rise to a discussion about the existence of a single system for estimating number, magnitude, space and time - the General Magnitude System (Lourenco & Longo, 2011). This general system became one of the main provisions of the Theory of Magnitude, which questioned the existence of a separate system for estimating discrete quantities (Walsh, 2003).

But if there is no separate system for numerosity estimation, and there is no special 'sensitivity' to quantity, then how can a person, for example, compare two sets of objects (which is possible, no one can argue with that)? To explain this, the theory of 'sensory integration' has been proposed (Gebuis et al., 2016). According to this theory, the estimation of the number of discrete objects is based on the estimation of several non-numerical visual parameters, such as the size of the shapes, their cumulative area, their density of arrangement and their surface area. Each visual parameter has its own 'weight' in the estimation of quantity (Clayton et al., 2015; Gilmore et al., 2016). In support of this theory, it has been found that changes in the activation of brain areas involved in quantity estimation are associated with changes in visual parameters rather than changes in their quantity (e.g. (Gebuis & Reynvoet, 2012).

It was also shown that the accuracy of the nonsymbolic comparison depended on the relationship between quantitative and visual parameters. When visual and quantitative parameters were positively correlated (e.g., a set containing a larger number of objects had a larger occupied area), i.e., congruent, accuracy was higher than in incongruent conditions (Clayton et al., 2015; Smets et al., 2016; Szűcs et al., 2013). This difference in accuracy between congruent tasks has been termed the congruency effect. It reflects the extent to which quantity estimates depend on visual parameter estimates. Furthermore, it has been shown that when sets are incongruent with respect to multiple visual parameters, comparison accuracy is critically reduced and does not exceed the random guessing threshold (Szűcs et al., 2013).

Viarouge et al. (2019) suggest that the congruency effect reflects the suppression of irrelevant cues, rather than the dependence of quantity estimation on visual parameter estimation. From this perspective, when comparing numerosities, participants can process both non-numerical visual parameters and quantitative features, but the former are processed faster.

The nonsymbolic number representation system may be the basis for the development of symbolic numerical skills (De Smedt et al., 2013). Symbolic and non-symbolic representations have been shown to share similar features, such as the Numerical Distance Effect (NDE) ((Halberda et al., 2008a; Holloway & Ansari, 2009) Holloway & Ansari, 2009). Another and more important argument comes from research on the relationship between non-symbolic number sense and mathematical achievement. A large number of studies have shown that the accuracy of non-symbolic number sense correlates with mathematical achievement measured at the same time or even several years later (Chen & Li, 2014; Libertus et al., 2012; Schneider et al., 2017).

However, some studies have found no significant relationship between non-symbolic number sense and symbolic math skills (Fuhs & McNeil, 2013; Gilmore et al., 2013). Some researchers believe that symbolic and non-symbolic number representation systems are two separate systems (Lyons et al., 2015; Sasanguie et al., 2017). For example, it has been shown that at the individual level there is no significant correlation between the effects of numerical proportion in number comparison and non-symbolic comparison tasks (Lyons et al., 2015).

In general, despite the growing number of studies on non-symbolic numerosity, there are several open questions. First of all, the mechanisms that enable the processing of non-symbolic quantitative information are still unclear. Is this system separate and independent from the estimation of non-numerical visual parameters? Can humans estimate quantities without relying on the estimation of continuous visual features? (e.g., de Hevia et al., 2017; Harvey & Dumoulin, 2017; Wilkey et al., 2017).

The second controversial debate concerns the extent to which non-symbolic number sense is related to symbolic numeracy. Can the accuracy of non-symbolic number sense predict mathematical achievement? Could the non-symbolic number representation system be a system that has been used during evolution to develop symbolic number skills? Researchers have attempted to answer these questions using a variety of approaches, both experimental (Park et al., 2016) and correlational (Halberda et al., 2008b). Neurophysiological and neuroimaging methods such as EEG (e.g. (Gebuis & Reynvoet, 2012)), fMRI (Mock et al., 2018), transcranial magnetic stimulation (Sasanguie et al., 2013) or eye tracking (e.g. (Price et al., 2017)) are also commonly used.

Each of these methods has its advantages and limitations. This review aims to analyse research on non-symbolic number sense using eye tracking.

Possibilities of using eye tracking to study numerosity processing

Eye tracking is an increasingly popular method for investigating the perception and processing of information, including quantitative information, both with and without the use of symbols (Hurst & Cordes, 2016; Irwin & Thomas, 2007; Lilienthal & Schindler, 2019;

Merkley & Ansari, 2010; Odic & Halberda, 2015; Price et al., 2017). Eye tracking provides direct access to internal processes by tracking the focus of attention with high spatial and temporal resolution.

Variables collected by recording eye movements include saccadic movements (movements of the eye that represent a change in the focus of attention) and fixations (maintaining perception in an area of interest and hence sustained concentration of attention). These indicators generally fall into two categories. The first category includes the location and duration of the first fixation and indicates bottom-up, stimulus-driven processes related to visual-perceptual processes and involuntary attention.

The second category reflects top-down processes related to voluntary control, attitude, and motivation, and involves more intensive and prolonged cognitive processing (Calvo & Meseguer, 2002; Mock et al., 2016). These include the total number of fixations, the total duration of fixations, and the frequency and direction of saccadic movements. Thus, based on the analysis of different types of oculomotor indicators, it is possible to say which processes - ascending or descending - are involved in the processing of quantitative information.

Characteristics of oculomotor responses can indicate which of the two visual systems, dorsal or ventral, is activated at a given time (Pannasch et al., 2008; Velichkovsky et al., 2005). Studies of visual perception have shown that morphological visual information can follow two pathways: dorsal and ventral (Mishkin et al., 1983). The ventral stream sends information from occipital regions to inferior temporal regions. Here, foveal information is processed at a relatively slow rate. The dorsal stream sends the signal faster, but with less spatial resolution, to posterior parietal regions.

The functional division is based on the processing of information in these streams in two modes: in the ventral stream, the subject stream, which answers the question 'what', and in the dorsal stream, the spatial stream, which answers the question 'where'. For example, comparing numbers has been shown to be a task that activates the dorsal pathway in the right parietal regions. However, the task of deciding whether a number is odd or even activates the ventral pathway (Klein & Knops, 2023). Thus, analysing oculomotor performance during different number operations can provide important information about the mental processes behind these operations.

A systematic review of eye tracking research in mathematics education has already been conducted (Strohmaier et al., 2020). It reviewed the main findings of research from 1921 to 2018, but its main focus was specifically on mathematics learning; this study did not include detailed findings of research on nonsymbolic representations of quantity, as not all number sense research is related to research on math skills.

Aims of the systematic review

Our systematic review includes eye tracking studies of a non-symbolic quantity representation system. The aims of this review are:

1. To highlight and describe the main research questions, instruments (tests, tasks) and results of research on the non-symbolic representation system using eye tracking;
2. To analyse the main ways of interpreting indicators of oculomotor reactions from the point of view of the characteristics of the non-symbolic representation system;
3. To identify the main mechanisms for processing nonsymbolic quantitative information based on the results of eye tracking studies.

Methods

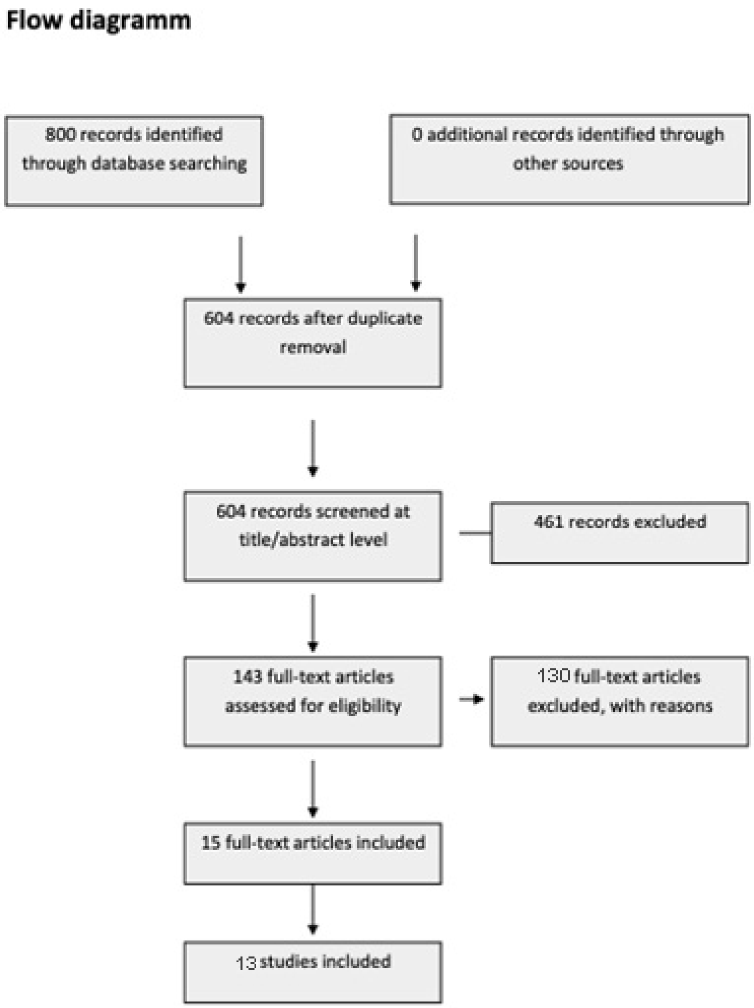
The systematic review was conducted in accordance with the Preferred Reporting Items for Systematic Reviews guidelines (Page et al., 2021). To identify the studies that met our criteria we implemented the following search strategy. The first step was the database search, for which we used the search string: ("approximate number sense*" OR "sense of magnitude*" OR "nonsymbolic comparison*" OR "symbolic representation*" OR "numerical representation*" OR "symbol grounding*" OR "number processing*" OR "representation of number magnitude*" OR "estimation biases*" OR "non-symbolic number comparison*" OR "non-symbolic representation*" OR "nonsymbolic representation*" OR "numerosity*" OR "numerosity processing*" OR "intuitive number sense*" OR "sense of number*") AND ("eye-movements*" OR "eye-tracking*" OR "saccade-terminated*" OR "eye-fixation behaviour*" OR "foveal*" OR “fixation*” OR “saccade*”), referring to titles and abstracts. Duplicates were automatically discarded. Articles were not limited by publication date. This resulted in a total of 604 studies.

Step two involved screening titles and abstracts for the inclusion: (a) the study was published in a journal article, a book chapter, or in conference proceedings; (b) the study was published in English; (c) the study involved eye-tracking; (d) the study used eye-tracking data to analyze non-symbolic numerosity estimation tasks. After this screening, 15 studies remained. We searched citation databases: Web of Science (n=59), Scopus (n=458), PubMed (n=87). Initial searches were conducted on October 06, 2023.

Citations were imported into CADIMA tool for systematic reviews (Kohl et al., 2018) for automated duplicate removal by title and screening. Titles and abstracts were screened by two reviewers (Sofia Mironets, Ilona Denisova) independently (Kappa value: 1). The two researchers agreed 92% of the time and discrepancies were resolved through discussions with the first author. CADIMA flowchart depicting the results of the selection process is shown in Fig. 1. The papers were assessed by two reviewers (SM, ID) independently; the consensus was reached in a group discussion.

Figure 1

CADIMA flowchart for study identification and selection



Results

Data collection

We reviewed 13 eye-tracking studies published between 2008 and 2023. The following information was extracted from the studies: (1) author, (2) title, (3) task type, (4) stimulus type, (5) stimulus presentation time, (6) sample characteristics, (7) oculomotor responses,

(8) equipment, (9) recording condition. Brief summaries of the 13 studies are outlined in chronological order in Table 1 for an overview (Appendix). All in all, there was a considerable overlap between the main variables in all empirical experiments as regards the examined samples, research procedure and equipment.

Main research questions

The 13 selected publications can be roughly categorized according to the research questions and aims. The most numerous group of publications focuses on studying the mechanisms of the system of nonsymbolic representation and its interactions with the system of processing space, time, depth, and other magnitudes. Six publications fall into this category (Bulf et al., 2016; Castaldi et al., 2020; Cheyette & Piantadosi, 2019; Lindskog et al., 2021; Odic and Halberda, 2015; Schütz, 2012).

Some of these papers are focused directly on the assessment of eyes movements, reflecting the distribution of attention resources when assessing the numerosity without the use of symbols. For example, Castaldi et al., 2020 investigates whether humans can choose the most numerous array of items with fast saccadic eye movements and how oculomotor movements differ depending on the number of objects being evaluated.

Other studies considered processing of numerosity, space and magnitudes in the framework of different theories which were mentioned in the introduction (e.g. theory of sensory integration or the ANS theory). Particularly, Odic and Halberda (2015) in the framework of the ANS theory considered the question if the patterns of eye-movement were different with the same stimulus regarding different tasks: comparison of numerosities and comparison of areas (magnitudes).

The second large category of selected papers included three papers, which focused on study of associations between symbolic and nonsymbolic numerosity representations (Guan et al., 2020; Peake et al., 2020; Price et al., 2017). It should be noted that in two papers (Peake et al., 2020; Price et al., 2017) it was considered the mechanisms of functioning of both symbolic and nonsymbolic representation and overlap between them. Hence, to some extent these papers may be also included in the first category. For example, Peake et al. (2020) considered how participants distributed their perceptual attention and focused on the stimuli in the process of symbolic and nonsymbolic comparison.

The next category included two papers, aimed to estimate differences in accuracy and eye-movement patterns between individuals with different disorders in comparison to typical developing individuals (Abreu-Mendoza et al., 2015; Van Herwegen et al., 2019). Particularly, Van Herwegen et al. (2019) considered eye-movement patterns during non-symbolic comparison for individuals with Down syndrome and Williams syndrome in comparison with individuals without such disorders.

Finally, two articles do not fall into any of the above categories. One study examines the process of approximate quantification, i.e. the rapid and approximate estimation of the number of objects by assigning numbers to sets of objects (Gandini et al., 2008).

This study identified different quantification strategies and investigated the patterns of oculomotor responses depending on the strategy. Another independent study aimed to investigate the contribution of genetic and environmental factors to some measures of non-symbolic number sense. In this case, eye tracking was only used to obtain one characteristic of the performance of the non-symbolic number representation system, namely gaze duration (Viktorsson et al., 2023).

In summary, a review of the main research questions revealed that eye-tracking research has been used to better understand the mechanisms of the functioning of the non-symbolic quantity representation system and its relationship to other numerosity or magnitude estimation systems, through analyses of oculomotor responses and allocation of attentional resources.

Task

Nonsymbolic comparison task

Nine of the thirteen selected papers used the classic nonsymbolic comparison task (NSCT), which can be subdivided into a Numerosity Comparison Task (NCT) and a Magnitude Comparison Task (MCT).

The Numerosity Comparison Task (sometimes called Number/Quantity Comparison Task) asks participants to estimate which of two arrays contains more objects. This task occurred in the 8 selected studies (Castaldi et al., 2020; Cheyette & Piantadosi, 2019; Guan et al., 2021; Lindskog et al., 2021; Odic & Halberda, 2015; Peake et al., 2020; Price et al., 2017; Van Herwegen et al., 2019). The most common stimuli were simple geometric shapes: dots and blobs (angular size ranged from 0.18 to 1.26°). Abreu-Mendoza et al. (2015) compared coloured cartoon images of food and animals.

In a Magnitude Comparison Task (also called Area Comparison Task), participants are asked to determine which of two arrays occupies the larger surface area (Odic & Halberda, 2015). Abreu-Mendoza et al. (2015) modified the task to match the stimulus material, asking participants to determine which of two cartoon images had more food.

Stimuli design

Stimuli in the non-symbolic comparison test can be either arrays with objects of the same colour (black, green, white) (Cheyette & Piantadosi, 2019; Guan et al., 2021; Price et al., 2017) or with objects of different colours (classically yellow and blue (Lindskog et al., 2021; Odic & Halberda, 2015); black and white (Castaldi et al., 2020); red and blue (Van Herwegen et al., 2019).

In all but one study (Cheyette & Piantadosi, 2019), the arrays for comparison were presented simultaneously on the screen in a separate format, with each array on the left or right side of the screen. In Cheyette & Piantadosi (2019), the arrays to be compared were presented sequentially, with no interstimulus interval.

The number of objects in the comparison arrays varied across studies. In general, the arrays contained between 4 and 90 objects. The most common values were between 5 and 16 objects (in each array). The number of stimuli used in each study is shown in Table 1.

Most studies controlled for the ratio of the number of objects in the arrays, defined as the lower number of objects divided by the higher number of objects. Only three papers included arrays with a low ratio of less than 0.5 (Abreu-Mendoza & Arias-Trejo, 2015; Castaldi et al., 2020; Peake et al., 2020). The most common ratio between arrays for comparison was 0.5 (found in eight studies), while ratios greater than 0.5 (0.6-0.91) were used in four studies (Castaldi et al., 2020).

Some papers assessed the effect of congruency by controlling the size ratio, total surface area, total cumulative area and density of objects in the array (e.g. (Lindskog et al., 2021). Dots could either have a predetermined size or vary in size depending on the above factors.

Presentation time

The duration of the array presentation depends on the research question and the oculometric parameters to be analysed. For example, Castaldi et al (2020) investigated fast saccadic movements and presented stimuli for 200 ms. In one of their experiments, Cheyette & Piantadosi (2019) manipulated the presentation duration of two arrays for comparison: dot sets could be presented for the same duration (100:100 ms, 1000:1000 ms) or for different durations (0:1000 ms, 1000:100 ms) to assess the role of the foveation accumulation effect.

Other studies have used presentation durations in the range of 1000 to 2000 ms, which is considered sufficient to produce several reliably recorded fixations.

Array estimation (enumeration) task

Two studies (Cheyette & Piantadosi, 2019; Gandini et al., 2008) used the Array Estimation/ Enumeration Task to estimate the number of points in an array.

Stimuli design

The selected papers used monochrome stimuli (black in (Gandini et al., 2008); blue in (Cheyette & Piantadosi, 2019). The size of the dots varied (Gandini et al., 2008 - 18px; Cheyette & Piantadosi, 2019 - 10px), as did the size of the sets to be compared. In the study by Gandini et al. (2008), target arrays consisted of 15, 20 or 25 dots, and control arrays could contain between 4 and 79 dots. The Cheyette & Piantadosi (2019) study contained between 10 and 90 dots.

Gandini et al. (2008) used a more complex experimental design in which participants were presented with black dots in cells of white grids. Thus, in contrast to the other two studies, the stimuli were ordered by fitting into the cells. In addition, the dots could be arranged both chaotically within the grids and in predetermined 'canonical' patterns.

Presentation time

The choice of stimulus presentation duration was related to the research questions. Cheyette & Piantadosi (2019) varied stimulus presentation durations (100, 333, 1000, 3000 ms) to assess the relevance of the number of fixations and foveations on the accuracy of object numerosity estimation. The stimuli in Gandini et al. (2008) remained on the screen for 6 s, during which time participants had to estimate numerosity using one of two strategies: perceptual estimation or anchor estimation. In the latter, participants were asked to count the dots in one of the clusters aloud and to estimate the number of remaining dots according to the experience gained in the first count.

Passive viewing tests

Two studies investigating non-symbolic number sense in infants used a passive viewing paradigm (Bulf et al., 2016; Viktorsson et al., 2023). In this paradigm, no action is required; the participant simply observes changes in the presented arrays of objects. Depending on the type of task, participants may be presented with two arrays of objects, one with a constant number of objects and the second with a changing number of objects (visual detection task). In this case, the average duration of gaze on the side with the changing number of dots was estimated. In the other type of task, arrays of objects or geometric shapes were used as 'cues' and presented before the target stimulus to determine the extent to which quantity or physical size could be a feature that determines the direction of attention (paired visual preference paradigm).

Stimuli design

Viktorsson et al (2022) showed infants a series of pictures with two dot arrays. The array on one side of the screen was numerically constant, while the array on the other side varied in the number of dots at a ratio of 1:1 and 1:2 or 1:1 and 1:4 to the constant array. The constant set consisted of 10 or 6 dots and the alternating set could contain from 6 to 24 dots.

In the study by Bulf et al. (2016), the arrays contained 2-9 dots. The task contained both congruent and incongruent trials. Congruency was determined by matching the side of the screen on which the 'cue' (a larger array or physically sized figure) and the target stimulus were displayed. Time to fixation on the target was assessed in the task.

Presentation time

The on-screen duration of the arrays was 500 ms in Viktorsson et al. (2022) and 300 ms in Bulf et al. (2016). The interstimulus interval was 300 ms (Viktorsson et al., 2023) and 400 ms (Bulf et al., 2016).

Sample

In the majority of studies, the sample consisted of healthy adults, most often students. Group sizes ranged from small (9-15 participants) (Castaldi et al., 2020; Gandini et al., 2008; Odic & Halberda, 2015) to medium (27-58 participants) (Cheyette & Piantadosi, 2019; Guan et al., 2021; Lindskog et al., 2021; Peake et al., 2020; Price et al., 2017).

The studies that examined differences between clinical and non-clinical samples also had small sample sizes and included participants of different ages (children, adolescents, adults). Group sizes ranged from 16 to 24 participants (Abreu-Mendoza & Arias-Trejo, 2015; Van Herwegen et al., 2019).

The largest sample size was reported in a study of young children. In the study by Viktorsson et al. (2022), the sample consisted of 514 twins (age 5 months). The sample in Bulf et al. (2016) consisted of 36 infants (age 8-9 months).

Discussion

In this section, we review the main findings from eye tracking studies of non-symbolic number sense according to the main research questions.

Mechanisms of the system of non-symbolic representation of quantity and its relation to the systems of estimation of continuous quantities

Results from eye-tracking studies of infants' perception of quantity suggest that the processing of quantity information is an automatic, bottom-up and, at least in part, biologically determined process (Bulf et al., 2016). Six-month-old infants already showed sensitivity to quantity: they looked longer at arrays with more objects, and no relationship was found between mean gaze duration and accuracy (Viktorsson et al., 2023).

It has also been found that even in infancy there is a link between the representation of quantity and space, with larger quantities being associated with the right side of space and smaller quantities with the left. As in adults, this association appears to be automatic in infants: numerical information elicits spontaneous shifts of visual attention to specific regions of space in a magnitude-dependent manner (Bulf et al., 2016). This suggests that the link between numerical order and left-right orientation emerges early in life, before the acquisition of symbolic knowledge. The involvement of spatial attention mechanisms in determining the number of objects may suggest that estimating quantity involves estimating the spatial location of objects.

The existence of a special sensitivity to quantity and the fact that the processing of quantitative information takes place at an automatic level also follows from estimates of the direction and duration of the first fixation. Saccadic movements have been shown to be controlled by feature salience: the most salient object (in this case, an array containing a larger number of objects) is more likely to be selected first (Lindskog et al., 2021; Peake et al., 2020). In addition, the duration of the first fixation is also longer for an array containing a larger number of objects (Peake et al., 2020).

The importance of quantitative features is further supported by the fact that the duration of the first fixation was longer for arrays with more complex quantitative proportions (Peake et al., 2020). The probability that the longest fixation was on a larger array also increased with increasing proportion, which is more consistent with the hypothesis of automatic processing of quantitative features.

At the same time, a study showed that the direction of attention is determined by the physical size of objects rather than their number. Specifically, gaze was directed to an array containing a larger number of objects only in congruent tasks (Lindskog et al., 2021). In contrast, in non-congruent tasks, initial gaze was more likely to be directed to an array containing fewer objects but with a larger cumulative area. This may suggest that the physical dimensions of objects are processed more automatically than quantitative parameters. It should be noted, however, that this does not rule out the existence of a separate process for estimating quantity.

The existence of a separate process for processing quantitative information was confirmed in a study by Odic & Halberda (2015), who demonstrated differences in oculomotor movements in the numerosity and area comparison tasks using the same arrays. For example, participants made faster saccades and switches between regions of interest when performing NCT vs. ACT. In addition, the number of saccades also increased in the condition with more complex numerosity ratios between the arrays being compared (Odic & Halberda, 2015). This increase may indicate a focal information processing stage, which is necessary to obtain a more detailed view of the perceived arrays. The encoding of information about the surface of the arrays was more dependent on distributed attention, as reflected in longer and less frequent saccades, less frequent switching, and a higher percentage of fixations in the centre of the screen. This suggests the existence of a distinct quantity estimation process that adapts to different contexts and is robust to changes in cumulative area or other visual parameters.

The importance of the number of switches for improving non-symbolic comparison accuracy is presented in the 'foveal accumulation' model (Cheyette & Piantadosi, 2019). According to this model, quantity estimation accuracy 'accumulates' as a series of visual fixations are made. The authors showed that estimation accuracy increases as the number of objects entering the visual field increases, with a smaller contribution from the peripheral dots. The proposed model suggests that numerosity estimation is closely related to the mechanisms that control eye movements and attention.

However, it should be noted that Lindskog et al. (2021) and Castaldi et al. (2020) presented arrays for comparison simultaneously, whereas in Cheyette & Piantadosi (2019) the arrays were presented sequentially. This format of stimulus presentation involves working memory and may lead to adjustments in the visual strategies used to estimate quantity. Sequential presentation allows the size of each array to be estimated separately, while in the simultaneous format the estimate is driven by the most salient feature of the objects, which could be their size or area. Thus, the proposed model of sequential foveation accumulation may contribute to the understanding of how the processes of quantity estimation and attentional allocation are related.

Eye tracking studies have also shown differences between the three systems of quantity representation (sabitising, non-symbolic number sense, and texture (density) estimation). In particular, a study by Castaldi et al. (2020) showed that saccade duration was shorter when comparing two arrays containing an 'average' number of objects (12 to 35) than when comparing a small number (up to 4) or a very large number (more than 100).

In conclusion, eye-tracking studies have shown that there is a specific mechanism for numerosity estimation that is separate from the estimation of other visual parameters. Furthermore, the link between numerosity estimation and the spatial distribution of attention has been demonstrated.

Relationship between symbolic and non-symbolic quantity representation systems

Investigating the relationship between symbolic and non-symbolic systems of quantity representation may contribute to ideas about the formation and development of mathematical ability. To assess the extent to which symbolic and non-symbolic representations of quantity share a common mechanism, differences in eye movement patterns in symbolic and non-symbolic comparison tasks were assessed (Peake et al., 2020; Price et al., 2017). As in previous studies, the first fixation was shown to occur at larger magnitudes, confirming that quantity estimation is driven by bottom-up attention, regardless of format (non-symbolic or symbolic). It is suggested that some aspects of the visual-perceptual processes underlying magnitude comparison are common to all formats and are related to the speed, but not the accuracy, of decisions.

However, specific visual-perceptual processing differed when comparing arrays of objects and numbers. Longer fixations were found when comparing arrays of objects than when comparing numbers. Furthermore, the effect of numerical proportion was more pronounced for both duration and number of fixations in the non-symbolic comparison (Guan et al., 2021; Price et al., 2017). However, in the study by Peak et al. (2020), the increased proportion effect for the non-symbolic comparison task (compared to numerical comparison) was only confirmed for reaction time, but not for the duration of the first fixation.

Differences between symbolic and non-symbolic quantity representation mechanisms were also evident in how maths anxiety altered the numerical proportion effect for both types of representation (Guan et al., 2021). In a non-symbolic comparison task, participants with high levels of math anxiety showed a larger proportion effect, as manifested by an increase in fixation duration when comparing arrays with a higher numerical proportion. Furthermore, this effect was enhanced in the presence of interfering information. For the two-digit comparison task, participants with higher levels of anxiety showed an increase in fixation duration in the presence of interfering information, but the numerical proportion effect did not vary significantly with level of maths anxiety.

Thus, studies on the relationship between non-symbolic and symbolic representation of quantity using eye tracking have shown the existence of specific mechanisms for each type of representation. First of all, these differences are manifested in the expression of the numerical proportion effect for two indicators of oculomotor responses: the duration of the first fixation and the number of fixations. For both types of quantitative representation, the duration of the first fixation was longer in the higher numerical proportion conditions than in the lower numerical proportion conditions. However, the differences were significantly greater for the non-symbolic comparison task.

The effect of different disorders on the representation of quantity

Studies of the oculomotor response in children with different types of disabilities have attempted to understand the nature of difficulties in mathematics acquisition in these children. Previous studies have suggested that impaired mathematics ability in children with Williams syndrome and Down syndrome is caused by atypical viewing patterns (Van Herwegen et al., 2019, 2020). However, the study by Van Herwegen et al. (2019) did not support this hypothesis. A comparison of several indicators of oculomotor responses (mean fixation rate, mean fixation duration, time to first fixation or duration of first gaze) during a non-symbolic comparison task showed no significant differences between the groups with disorders and controls.

A similar result was found in a study comparing number and area in children with Down syndrome compared to typically developing children (Abreu-Mendoza & Arias-Trejo, 2015). The researchers calculated the difference in fixation duration between a target stimulus (e.g., an array with more objects) and a distractor (an array with fewer objects). A positive value of this index indicates a preference for the target over the distractor, while a negative value indicates the opposite. In addition, this index also indicates the duration of information processing when comparing quantities and areas.

In general, children with Down syndrome showed the same pattern of task performance as control children of comparable mental age. First, children with Down syndrome were more successful at comparing areas than at comparing quantities, i.e. the difference in gaze duration between the larger and smaller stimuli was greater for the area comparison task. Second, children with Down syndrome also showed a significant numerical proportion effect - a reduction in the difference in first look duration as the numerical proportion between the quantities being compared increased. Importantly, the numerical proportion effect was not significantly different between children with Down syndrome and the control group.

For example, studies have shown the preservation of non-symbolic comparison processes in children with Down syndrome and Williams syndrome, and the similarity of oculomotor responses to non-symbolic comparison in children with disabilities and typically developing children. This may suggest that the basis of the difficulties in mastering mathematics in children with disorders does not lie in disorders of the system of non-symbolic representation of quantity.

Approximate quantification mechanisms

One study has been devoted to investigating the mechanisms of approximate quantification, which we will focus on in more detail. Gandini et al (2008) identified 5 main quantification strategies and examined the patterns of oculomotor responses depending on the strategy. The main strategies are the anchoring strategy, the benchmark strategy, the decomposition strategy, the approximate counting strategy and the exact counting strategy. An anchoring strategy involves, for example, a participant counting a number of dots and then visually estimating the remaining number of dots by comparing it to a subset that has already been counted. The benchmark strategy involves participants comparing the stimulus with a representation held in long-term memory and then adjusting their response based on the estimated difference. The accuracy of quantity estimation differed according to the strategy used. Participants were less accurate but faster when using the benchmark strategy than when using the fixation strategy.

It should be noted that the choice of strategy depends on the stimulus configuration (random or non-random configuration), the number of objects to judge and the age of the participants. For example, the approximate strategy was used more often when estimating the number of dots in the random configuration and when increasing the number of dots in the estimated array. For younger participants the most popular strategies were approximate counting and benchmarking, and for older participants the most popular strategies were approximate counting and exact counting.

Oculomotor performance was used as a more precise measure of cognitive processes within each strategy. It was shown that the mean number of fixations and saccade amplitude differed depending on the strategy used and the time interval from stimulus presentation (in the first 500 ms of comparison and from 500 to 1000 ms). Saccade amplitudes were larger between 500 and 100 ms of task performance than in the first interval up to 500 ms for the benchmark strategy, but not for the anchoring strategy. Thus, when participants used the benchmark strategy, they made small amplitude saccades first, followed by larger saccades. In contrast, when participants used the anchoring strategy, they made larger amplitude saccades in the first interval than in the second interval.

In conclusion, the results of the study show that eye movements during approximate quantity estimation (number of fixations, saccade amplitudes) are sensitive to stimulus features (in particular their number and spatial position) and also depend on the strategy and the age of the participants. This study is the first to directly demonstrate that people use a wide range of approximate quantity estimation strategies in addition to basing their estimates on the visual properties of stimuli.

Conclusion

The recording of eye movements is becoming increasingly popular in the study of the perception and processing of quantitative information. Eye tracking provides insight into the mechanisms of perceptual and behavioural processes involved in the processing of quantitative information. This systematic review examined eye-tracking studies of non-symbolic quantity representation processes published in the last 15 years (from 2008 to 2023). Thirteen studies met the selection criteria and were grouped according to the main research questions. Most studies focused on investigating the mechanisms of non-symbolic numerosity representation and assessing the relationship between numerosity estimation and the estimation of non-numerical visual parameters.

Researchers have used different metrics for oculomotor responses, which sometimes makes it difficult to compare results. One of the most commonly used measures is the location of the first gaze. Most studies have shown that participants tend to direct their initial gaze to a set containing a larger number of objects or a figure with a larger area. This supports the hypothesis that quantity processing is a bottom-up process and that quantity is a visual feature that is processed at the level of precognition.

Number sense studies have also looked at indicators such as fixation duration and number of saccades. It has been shown that the number of saccades increases and the duration of fixations decreases with increasing cognitive load associated with increasing numerical proportion between the compared object arrays. This may be due to the involvement of the focal attention system with increasing cognitive load.

The results suggest that there is a separate process for processing quantity information, independent of the evaluation of other visual parameters. Eye-tracking studies have shown specific features of oculomotor responses in quantity comparison tasks, depending on both stimulus characteristics (e.g. number, numerical proportion, congruence) and respondent characteristics.

In general, it can be said that the processing of quantitative information can occur directly at the moment the information enters the visual system. In the first stage, there is an initial 'coarse' processing of quantitative information based more on low-frequency information, resulting in a kind of topographically organised map of perceived objects. In the second stage, this initial information is refined by processing high-frequency information (e.g. (Fornaciai & Park, 2021).

However, it must be recognised that eye-tracking studies have focused less on the stages of processing quantitative information and forming a representation of quantity. Most studies have been limited to comparing different indicators of eye movements for different types of tasks and stimuli. An exception is the work of Gandini et al. (2008), who investigated the temporal dynamics of eye movement characteristics in quantity estimation tasks. It is possible that a more detailed study of the temporal changes in indicators such as the number and duration of fixations, amplitude and number of saccades during the performance of quantity comparison tasks will provide a more detailed picture of how the internal representation of quantity is formed.

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Conflict of Interest Information

The authors have no conflicts of interest to declare.

Appendix A

Table 1
A brief summary of the studies

Authors	Ti
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itle	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
's more ind it: ed depth biases ceived erosity/ nsity	Determine direction and number comparison	Black and white dots on a grey background; the dots moved at a speed of 10 deg/s and had a limited presentation duration of 200 ms. 7 experimental tasks with different stimulus speed and brightness conditions	0-600 ms	12 (4 male; 8 female); aged 20-31 years	smooth pursuit eye movements, eye velocity; the eye velocity in the orthogonal direction	EyeLink; 1000 Hz	Monitor distance 47 cm

Authors	Title	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
Bulf et al., 2016	Small on the left, large on the right: numbers orient visual attention onto space in preverbal infants	Number comparison. Area comparison	The visual target appeared either to the left or to the right of the screen immediately after the appearance of a centred image of either small or large size. Both tasks consisted of a small number of objects (e.g. 2 dots) or a large number of objects (e.g. 9 dots); the 60 trials were divided into three blocks. Each block consisted of 16 experimental trials and 4 control trials. In total, there were 48 experimental trials.	100-2000 ms	36 infants	Three areas of interest: accuracy, target stimulus fixation time	an ASL6 remote eye-tracking system; 120 Hz	Car seat; monitor distance 60 cm

Authors	Title	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
Price et al., 2017	Eye-movement patterns during nonsymbolic and symbolic numerical magnitude comparison and their relation to math calculation skills	Number comparison	Two sets of dots were presented simultaneously on either side of the central fixation point. The black dot sets ranged from 6 to 15 dots on a white background. For analysis, a total of 72 trials were divided into 36 small ratio trials (ratio < 0.7) and 36 large ratio trials (ratio > 0.7), for a total of 14 different ratios.	1000 ms	56 (36 female); average age 19.4 years	Number of fixations; duration of fixation; spatial distribution of fixations; proportion of first fixations at higher numerosity; number of fixations at correct and incorrect numerosity (symbolic; non-symbolic).	EyeLink; 1000 Hz	Frontal chin support, monitor distance 58 cm

Authors	Title	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
Van Herwegen et al., 2019	Eye Movement Patterns and Approximate Number Sense Task Performance in Williams Syndrome and Down Syndrome: A Developmental Perspective	Number comparison	Congruent / incongruent (see similar task in Van Herwegen et al., 2018)	1500 ms	Two clinical groups: 24 (18 women); 25 (11 women); two control groups: 24 (12 women); 24 (17 women)	Time before start of viewing; mean fixation duration; mean proportion of viewing; duration of first fixation, total number of fixations; averaged over the left and right edges of the screen; differences in eye movements between groups.	Tobii T120 screen-based eye tracker; 120 Hz	Monitor distance 60 cm

Authors	Title	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
Peake et al., 2020	First fixation duration as a bottom-up measure during symbolic and non-symbolic numerical comparisons	Symbolic and non-symbolic number comparison	Stimulus set size ranged from 1 to 9 in the symbolic task and from 4 to 15 in the non-symbolic task. Each task consisted of 72 trials	-	32 participants, evenly distributed by gender; average age 20 years	Duration of first fixation	Eyelink II eye tracker (SR Research, Kanata, Ontario, Canada); 500 Hz	Monitor distance 60 cm

Authors	Title	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
Castaldi et al., 2020	Fast saccadic eye-movements in humans suggest that numerosity perception is automatic and direct	Number comparison	Arrays of white and black dots on a medium grey background, bounded by a circle. Number of dots: 1-4 dots - Subitising range; 12, 17, 24, 35 dots - Estimating range; 158, 195, 240 and 296 dots - Texture density range. 96 trials	200 ms	Experiment 1: 14 (6 male); Experiment 2: 11 (5 male)	Saccades; saccade direction; corrective saccade; reaction time (ms, min/max); correct response	an infrared eye tracker (EyeLink; 1000 Hz)	-

Authors	Title	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
Lindskog et al., 2021	Attentional bias induced by stimulus control (ABC) impairs measures of the approximate number system	Number comparison	90 images containing two spatially separated arrays of black dots on a white background. Half of the images contained 7 and 8 dots in the two arrays, while the other half contained 14 and 16 dots. One third of the images were congruent, incongruent and neutral, with no systematic difference in dot size between the two arrays.	1000 ms	40 (23 female); average age 24 years	Three areas of interest: fixation time; first fixation to area	Tobii T120 (Stockholm, Sweden); 60 Hz	Participants were instructed to remain as still as possible throughout the procedure

Authors	Title	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
Viktorsson et al., 2022	Infants' sense of approximate numerosity: Heritability and link to other concurrent traits	Quantity change detection	The videos consisted of a series of images, with two sets of dots appearing on the left and right sides of the screen. On one side of the screen the set of dots was numerically constant, while on the other side the set of dots alternated in number (10 and 20 or 6 and 24 dots). Each condition consisted of four stimulus videos.	500 ms	514 same-sex twins in infancy	Average viewing time on the side changing numerically (in relation to the whole screen), expressed as a percentage	Tobii T 120; 60 Hz	Child sitting on parent's lap; monitor distance 65 cm

Authors	Title	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
Abreu-Mendoza et al., 2015	Numerical and area comparison abilities in Down syndrome	Cartoon pictures area comparison Number comparison	The visual stimuli were six coloured cartoon pictures of larger and smaller food items. For the number comparison task, the visual stimuli were six coloured cartoon images of animals.	2500 ms	clinical group: 16 (7 females) and two control groups of 16 each	Two regions of interest: one for the target and one for the distractor. The difference in gaze duration (Schafer & Plunkett, 1998), which is the difference between a single sustained look at the target and a single sustained look at the distractor.	A portable eye-tracker (Tobii X2-30); 30 Hz	Monitor distance 60 cm

Authors	Title	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
Gandini et al., 2008	Older and younger adults' strategies in approximate quantification	Number estimation (naming without counting)	144 configurations of black dots presented as a visible square grid on a white background. Two thirds of these were experimental stimuli (including 15, 20 or 25 dots). The set of 144 gratings was divided into three series of 48 trials each	6000 ms	15 (7 female); mean age 26.8 and 15 (9 female); mean age 69.8	Mean number of fixations and mean saccade amplitudes; strategy	iView Remote Eyetracking Device (SensoMotoric Instruments); 50 Hz	Not to make sudden movements of the head or body, but no device restricted their movements; monitor distance 60 cm

Authors	Title	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
Odic & Halberda, 2015	Eye movements reveal distinct encoding patterns for number and cumulative surface area in random dot arrays	Number comparison. Area comparison	The stimulus images used in the two tasks were identical and consisted of multiple blue and yellow dots	2000 ms	12 adults	The onset of the first saccade; the proportion of time spent looking at each region of interest; the location and duration of the first, last and longest fixation; and the number of switches between regions of interest. Pupil size	Tobii TX300; 300 Hz	-
Guan et al., 2020	Non-symbolic representation is modulated by math anxiety and cognitive inhibition while symbolic representation not	Number comparison	A total of 40 trials, two sets of 20 trials each: a set of dots with a larger number and a larger area had to be compared with a set of dots with a smaller number and a smaller area; a set of dots with a larger number but a smaller area was compared with a set of dots with a smaller number but a larger area.	more than 2000 ms before response	19 students with HMA (74% female) and 16 students with LMA (88% female)	Area of interest: number of fixations and duration of fixations	Eyelink; 1000 Гц (SR Research, Mississauga, Ontario, Canada)	Frontal chin support

Authors	Title	Task type	Type of stimulus. Colour, shape, presentation format, quantity, number of presentations	Stimulus presentation time	Sample characteristics (n)	Oculomotor indicators	Equipment, model (type)	Recording conditions
Cheyette & Piantadosi, 2019	A primarily serial, foveal accumulator underlies approximate numerical estimation	Number estimation (naming without counting). Number comparison	An array of blue dots (10 to 90) on a white background, which were masked by noise after a short time in two experiments. Experiment 1 consisted of 64 trials consisting of 4 blocks of 16 trials each. The 4 different temporal conditions were: 100; 333; 1000 and 3000 ms. Experiment 2: two dot flashes, one after the other of the same stimuli, 4 conditions of 16 trials each (as in Experiment 1), 4 conditions (100; 100 ms), (100; 1000 ms), (1000; 100 ms) and (1000; 1000 ms).	100-3000 ms	27 adults (15 female); mean age 21.4 years	Fixing positions	Tobii T60XL; 60 Гц	Frontal chin support, monitor distance 66,4 cm